High-order regularization in machine learning and learning-based control

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1 Introduction

Regularization in machine learning is used to improve the generalizability of a model. A regularization method imposes penalties on some properties of the model to avoid overfitting the training data and allow for better generalization to the unseen test data [1]. The penalty terms can be designed to limit the complexity of a model and obtain a simple regularized model with similar or even higher performance. Some examples of overfitting models (blue line) and regularized models (black line) in two classical problems (regression and classification) in machine learning are presented in Figure 1.

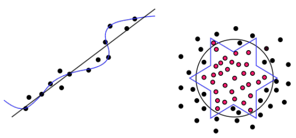

Figure 1: Regression problems (left) and classification problems (right) in machine learning.

2 The extreme learning machine and learning-based control

The extreme learning machine (ELM) is a machine learning algorithm. The original ELM is a special single-hidden layer feedforward neural network (SLFN) whose hidden nodes are randomly chosen and the output weights are analytically determined[2]. The architecture of ELM is shown in Figure 2. With the weights ω and the (possible) bias b from the input layer to the hidden layer randomly initialed, the estimation of the weight β from the hidden layer to the output layer of the ELM is determined by a least square method as follows:

$$\hat{\beta} = H^{\dagger} Y \tag{1}$$

where $H^{\dagger}$ is the Moore-Penrose inverse of the hidden layer output matrix H, and Y represents the output data in the training dataset. Extreme Q-learning machine (EQLM) is an approach that alters the gradient-based updates in a conventional reinforcement learning (RL) algorithm to optimize weights for a specific problem using ELM [3]. Therefore, one can improve the performance of EQLM by mitigating the overfitting problem in ELM.

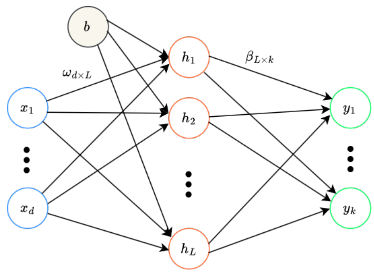

Figure 2: The ELM architecture.

3 High-order regularization in ELM

To mitigate the overfitting of ELM, our approach is to determine the output weights using a high-order regularization method, which is shown as follows:

$$\hat{\beta}_{HR} = \left(H^T H + R\right)^{-1} \sum_{i=0}^{k} \left(R\left(H^T H + R\right)^{-1}\right)^{i} H^T Y \tag{2}$$

where R is a regularization matrix, and k is a hyperparameter with $k = 1$ in the most usual case.

4 Conclusion

In this work, we proposed a new extreme learning machine with the high-order regularization method to improve the behaviors of the extreme learning machine. High-order regularization is used to calculate the output weights, which link the hidden layer and the output in the ELM network.

References

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[3] Callum Wilson, Annalisa Riccardi, and Edmondo Minisci. A novel update mechanism for Q-networks based on extreme learning machines. In *2020 International Joint Conference on Neural Networks (IJCNN)*, pages 1–7, 2020.